

1. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geq 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

- (b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

- (c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

- (d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

- (e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

- (f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

$$(a) P(X \geq 16) = 1 - P(X \leq 15)$$

$$= 1 - 0.9491$$

$$= \boxed{0.0509 \text{ (to 4 d.p.)}}$$

$$(b) H_0: p = 0.3, H_1: p \neq 0.3$$

(c) Let y be the number of customers who buy tins out of a sample of 20.

$$y \sim B(20, 0.3)$$

$$P(Y \leq 3) = 0.1071 > 0.05$$

$$P(Y \leq 2) = 0.0355 < 0.05$$

Question continued

Critical region for the lower tail is $y \leq 2$

$$\begin{aligned} P(X \geq 9) &= 1 - P(X \leq 8) \\ &= 1 - 0.8867 \\ &= \underline{0.1133} > 0.05 \end{aligned}$$

$$\begin{aligned} P(Y \geq 10) &= 1 - P(Y \leq 9) \\ &= 1 - 0.9520 \\ &= \underline{0.0480} < 0.05 \end{aligned}$$

Critical region for the upper tail is $y \geq 10$

\therefore Critical region is $y \leq 2$ or $y \geq 10$

(d) Actual significance level = $0.0355 + 0.0480$

$$= \underline{0.0835}$$

(e) Since the observed value of 12 lies within the critical region, there is sufficient evidence to reject H_0 : the manager's belief is supported.

(f) If the visitors came as a scout group, this means that it wasn't a random sample taken and also each tin of baked beans wasn't bought independently. This invalidates the use of a binomial distribution as a model and thus the answer in part (e) too.

(Total for Question is 9 marks)

TOTAL FOR SECTION A IS 30 MARKS

2. Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$. Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

- (i) exactly 2 games,
(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

Let $X =$ no. of games Naasir wins

$$X \sim B(15, \frac{1}{3})$$

a i) $P(X=2) = 0.0599$

ii) $P(X > 5) = 1 - P(X \leq 5)$

$$= 1 - 0.61837$$

$$= 0.38162$$

$$\approx 0.382$$

b) $H_0: p = \frac{1}{3}$

$H_1: p > \frac{1}{3}$ 5% S.L.

$$X \sim B(32, \frac{1}{3})$$

$$P(X \geq 16) = 1 - P(X \leq 15)$$

$$= 1 - 0.96235$$

$$= 0.03765 < 0.05$$

\therefore Reject H_0 . The evidence shows that Naasir's claim is supported.



3. Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$$H_0 : p = 0.15$$

$$H_1 : p \geq 0.15$$

Let X = the number of customers who buy chocolate.

$$X \sim B(30, 0.15)$$

$$P(X = 8) = 0.0420$$

$$0.0420 < 0.05 \text{ so reject } H_0$$

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

- (a) Identify the first two errors that Julie has made in her hypothesis test. (2)
- (b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer. (1)
- (c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05 (2)
- (d) Find the actual level of significance of this test. (1)

a. H_1 should be: $p > 0.15$ rather than $p \geq 0.15$

Test statistic should be calculated by $P(X \geq 8)$, not $P(X = 8)$.

b. Yes, $P(X \geq 8) = 0.0698$, > 0.05 so this finding is in fact significant.

$$1 - P(X \leq 7) = 1 - 0.9302$$



Question continued

$$c. P(x \leq 8) = 0.9722 > 0.95$$

$$P(x \geq 9) = 0.0278 < 0.05$$

\Rightarrow critical region: $x \geq 9$

$$d. 0.0278, 2.78\%$$

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4. Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful. (2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days. (2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief. (4)

a) independent, constant probability of success \Rightarrow prompts us to use the binomial distribution.

let C = number of successful calls.

$$C \sim B\left(9, \frac{1}{6}\right)$$

\uparrow no. attempts
 \leftarrow probability

$$P(C \geq 3) = 1 - P(C \leq 2) = 0.1782... \text{ by calculator}$$

b) we use the value of $P(C \geq 3)$ from (a) \leftarrow imagine we

let X = the number of days when at least 'trial' the 9 calls on

3 calls succeed. 5 days, with a new p



Question continued

$$X \sim (5, P(c \geq 3))$$

$$P(X=1) = 5 \times (0.1782) \times (1-0.1782)^4 \\ = 0.4061\dots$$

c) 1. state your hypotheses

$$H_0: p = \frac{1}{6} \quad H_1: p > \frac{1}{6}$$

2. define your variables & calculate test statistic

let R = number of successful calls

$$R \sim B(35, \frac{1}{6})$$

$$P(R \geq 11) = 1 - P(R \leq 10) = 0.02\dots$$

3. form conclusion

$0.02 < 0.05 \Rightarrow$ there is sufficient evidence to support that

Rowan has more successful calls than Afrika. reject H_0 .

always link to context of test



5. A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

- (a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet. (1)
- (b) Use this binomial distribution to find the probability that
- (i) Aliya has just 1 red bead in her bracelet,
- (ii) there are at least 4 red beads in Aliya's bracelet. (3)
- (c) Comment on the suitability of a binomial distribution to model this situation. (1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed. She takes a random sample of 75 beads and finds 4 red beads.

- (d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher. (4)
- (e) Find the p -value in this case. (1)

(a) Let $R =$ number of red beads in Aliya's bracelet

$$R \sim B(18, 0.14) \quad * \quad (1)$$

(b)(i) $P(R=1) = 0.1940 \dots \quad (1)$

(ii) $P(R \geq 4) = 1 - P(R \leq 3)$

$$= 1 - 0.761 \dots \quad (1)$$

$$= 0.238 \dots \quad (1)$$

(c) Requires $p = 0.14$ to be constant so need large number of beads in the sack to ensure that removing 18 beads does not appreciably affect this probability, then it could be suitable. (1)



Question continued

(d) Let X = Number of red beads in the sample

$$X \sim B(75, 0.14) \quad (1)$$

$$E(X) = 75 \times 0.14 = 10.5$$

$$H_0 : p = 0.14 \quad , \quad H_1 : p \neq 0.14 \quad (1)$$

$$P(X \leq 4) = 0.01506 \dots \quad (1)$$

Since $0.01506 \dots < 0.025$, so we reject H_0 .

\therefore Hence, there is evidence to suggest proportion of red beads has changed. (1)

$$\begin{aligned} \text{(e) } p\text{-value} &= 2 \times 0.01506 \dots \\ &= 0.030123 \quad (1) \end{aligned}$$

